Model Specification in the Analysis of Spatial Dependence

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Abstract

The recent surge in studies analyzing spatial dependence in political science has gone hand in hand with increased attention paid to the choice of estimation technique. In comparison, specification choice has been largely neglected, even though it leads to equally, if not more, important inference problems. In this article we discuss four issues. Two pitfalls can simply be avoided if researchers adequately control for common trends and shocks and refrain from inferring anything on the validity of the chosen weighting matrix from the estimation results of the spatial lag. The two remaining specification choices have no easy solution, however. First, row-standardization of the weighting matrix changes the relative weight of observations that make up the spatial effect and thereby impacts on statistical inference of the spatial lag. It should therefore only be applied if theoretically warranted, not as a general default rule. Second, seemingly small changes to the functional form of the weighting matrix can change dramatically the coefficient sign, size and standard error of the spatial lag. This creates the temptation of choosing a functional form that generates results consistent with the hypothesis to be tested. Only a more developed theoretical model can defend researchers against verification bias by providing guidance on the functional form of the weighting matrix. We demonstrate the importance of all four specification issues by replicating a prominent model of spatial dependence in international capital tax rate competition.

1. Introduction

Political units often spatially depend on each other in their policy choices. For example, capital tax rates in one country are typically affected by tax policies in other countries. Similar patterns of spatial dependence have been studied in areas as diverse as social policies (Franzese and Hays 2006, Brooks 2007, Cho 2003, Bailey and Rom 2004; Jahn 2006), monetary policies (Simmons and Elkins 2004; Plümper and Troeger 2008), tax and fiscal policies (Basinger and Hallerberg 2004; Hays 2003, 2008; Swank 2006), trade and investment policies (Mansfield and Reinhardt 2003; Elkins, Guzman and Simmons 2006), military spending and armed conflict (Shin and Ward 1999; Salehyan and Gleditsch 2006), democratization (Gleditsch and Ward 2006), and many others.

A search through the top 50 (in terms of total cites) political science journals in the Social Sciences Citation Index revealed very few studies published in the 1990s that included spatial effects, but almost 50 articles already published in this decade. This surging interest in analyzing spatial dependence in the political sciences was fuelled by two developments: the rapid increase in global market integration, technological changes and cross-border communication on the one hand, and the rapid improvement in both computing power and spatial econometric methods on the other hand (Anselin 1988, Beck et al. 2006; Franzese and Hays 2007, 2008; Ward and Gleditsch 2008). While the first development raised the interest in spatial analyses, the latter facilitated the actual estimation of spatial dependencies by providing guidance on the choice of estimation technique.

This paper analyzes three important specification issues in spatial econometrics.[1] We demonstrate that these exert a relatively neglected, but equally, if not more, important influence on the estimation results. First, failure to control for common shocks and common trends in cross-sectional time-series or panel data is likely to lead to upward bias for the

[1] There are more issues of specification choice of course, which we cannot discuss here for reasons of space (see, for example, Darmofal 2006).

spatial effect. In addition, different, but equally justifiable specifications of the weighting matrix can easily lead to starkly differing results. This threatens the validity and reliability of inference. In particular, we show that row-standardization of the weighting matrix changes the relative influence of other units on the spatial effect and thereby changes the estimation results. Despite being regarded as usual practice by spatial econometricians, it is not always appropriate, requires theoretical justification and should therefore not be applied as a general default rule. Finally, changes to the functional form of the weighting matrix can dramatically change the estimated results of the spatial effect. This is of great importance because existing theories of spatial dependence typically do not derive a functional form for the connectivity or weighting matrix. This is amplified by the known fact that one cannot interpret estimation results on the spatial effect as evidence for the correct specification of the connectivity or weighting matrix.

We use one of Hays's (2003, 2008) models of tax competition for replication purposes. We have chosen his work not because his models are flawed (they are not), but because they represent the state of the art of empirical research into spatial dependence in political science. We demonstrate that model specification has a very large effect on the estimation results for the spatial effect. Whilst we show this for the specific results reported in Hays (2003, 2008), we contend that the non-robustness of results to small changes in model specification extends to virtually all studies of spatial dependence.

Researchers can easily control for common trends and shocks. There is, however, no easy remedy to the problem of specifying the weighting matrix. This opens spatial analysts to the charge that they can produce results that fit their hypotheses by making one or more seemingly arbitrary specification decisions. We offer two potential solutions to this problem. Ideally, scholars formulate their theories more comprehensively providing sufficient detail on the spatial effect modeling. Theory should be able to decide on the question of row-standardization. And while it will rarely be able to specify the exact functional form of the weighting matrix, it can often exclude certain functional forms. In the absence of sufficiently specified theories, the second-best solution is to show in robustness tests how the results on the spatial effect change if different functional forms of the weighting matrix are used.

2. Modeling Spatial Effects: a Very Brief Overview

There are three ways of modeling spatial effects, namely as spatial lag, spatial-x and spatial error models. Spatial lag models regress the dependent variable on the spatially lagged dependent variable, that is, on the (weighted) values of the very same dependent variable in all other units. In a monadic cross-sectional time-series or panel dataset, the spatial lag is formally modeled as follows:[2]

$$y_{it} = \alpha + \rho \mathbf{W} y_{-it} + ... + \varepsilon_{it} \quad \forall \; i \neq -i \quad . \tag{1}$$

where notation is standard so that y_{it} is the value of the dependent variable in unit i at time t, estimated with a spatially lagged dependent variable ($\mathbf{W}y_{-it}$) and an iid. error process ε_{it}. The spatial lag $\mathbf{W}y_{-it}$ consists of two elements, namely what we call the "spatial *y*" (y_{-it}) and the spatial weighting matrix $\mathbf{W}$.[3] The spatial *y* is the contemporaneous value of the dependent variable in all units *-i*, that is, all units other than *i.*[4] This is multiplied with an $N \cdot N$ block-diagonal spatial weighting matrix $\mathbf{W}$, which measures the relative connectivity between *N* number of units *i* and all other units *-i* $(i \neq -i)$ in the off-diagonal cells of the matrix (the diagonal of the matrix has values of zero). The spatial autoregression parameter ρ gives the impact of the spatial lag $\mathbf{W}y_{-it}$ on y_{it}. The dots represent other variables potentially included in the model such as control variables, unit fixed effects, period fixed effects, and variables

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2 The analysis of spatial dependence is more flexible but also more complicated in dyadic data – see Neumayer and Plümper (2008) for an analysis of all the possible forms of modeling spatial dependence in such datasets.

3 We would call y_i the spatially lagged dependent variable, which we regard as the more appropriate term, if Anselin (2003: 159) and others did not use this term for the entire spatial lag $\mathbf{W}y_{-it}$.

4 The spatial y may also be temporally lagged which can be advantageous for estimation purposes – see Beck et al. (2006) for details.

which account for the dynamics of the data generating process in a way which ensures that the Gauss-Markov conditions are satisfied.

Spatial-x models regress the dependent variable on the (weighted) values of one or more independent explanatory variables (other than the dependent variable) in all other units:

$$y_{it} = \alpha + \rho \mathbf{W} x_{-it} + ... + \varepsilon_{it} \quad \forall \;\; i \neq -i \quad . \tag{2}$$

Spatial error models seek to identify spatial dependence in the error term. These differ slightly from spatial lag and spatial-x models as they require a two-stage approach. In the first stage of the estimation, one estimates a model without spatial dependence, where it is assumed that $e_{it} = u_{it} + \varepsilon_{it}$ and where u_{it} is the spatially dependent part of the error process so that $cov(u_i, u_{-i}) \neq 0$. The second stage then estimates

$$y_{it} = \alpha + \rho \mathbf{W} \hat{u}_{-it} + ... + \varepsilon_{it} \quad \forall \;\; i \neq -i \quad . \tag{3}$$

Political scientists have focused much of their attention on the choice of estimation technique for these models.[5] Here we will instead discuss only specification issues, disregarding entirely the choice of estimator. We concentrate on spatial lag models, which are the most popular in political science, but everything we say applies similarly to spatial-x and spatial error models.

3. Dynamics, Common Trends, and Common Shocks

Most spatial lag models in political science use cross-sectional time-series or panel data, which have well known advantages over cross-sectional designs. Amongst other things, they allow accounting for dynamics, common trends and common shocks. At the same time, however, failure to control for these complications in the data generating process has even

[5] Based on Monte Carlo analyses, Franzese and Hays (2007) have demonstrated that Spatial-OLS, Spatial-2SLS and Spatial-ML provide flexible approaches to estimating different types of spatial dependencies. For example, using OLS as an estimator of spatial dependence (spatial-OLS) works well if researchers either analyze spatial-x models or spatial lag models with sender-receiver relations in which senders cannot also be receivers. Using a maximum likelihood estimator (spatial-ML) instead usually changes results only marginally. If, however, in a spatial lag model the sender can also be a receiver, researchers need to solve the endogeneity problem, which can be done by using instruments for the spatially dependent variable (spatial-2SLS).

more severe consequences in spatial than in standard panel data analysis. Such failure will inevitably lead to upward biased spatial effects and may thus cause wrong inferences. Even though this problem is widely discussed in the theoretical literature in spatial econometrics (e.g., Beck et al. 2006; Franzese and Hays 2006), only a minority of analyses control for common trends by adding the lagged dependent variable to the list of regressors (e.g., Hays 2003, Franzese and Hays 2006, Swank 2006) or additionally account for common shocks by further adding period dummies (e.g. Bailey and Rom 2004; Madariaga and Poncet 2007; Franzese and Hays 2006; Hays 2008).

To demonstrate the effect of failing to model dynamics and control for common shocks and trends, we analyze the case of capital taxation in OECD countries. Theories of tax competition contend that when capital is fully or partially mobile, independent jurisdictions compete to some extent for a common tax base (Wildasin 1989; Plümper et al 2007). The lower the effective tax rate in one jurisdiction relative to those of other jurisdictions, the larger the share of the mobile tax base it will attract. Thus, low capital taxation leads to an inflow of capital, which at least in the short run increases the tax base of the capital importing jurisdiction so that tax revenue may increase even though the tax rate becomes smaller.

Yet, the success of one jurisdiction in attracting mobile capital leads to a decline in the tax revenue for the other jurisdictions. If policy-makers in these jurisdictions want to avoid budget deficits, they either need to increase taxes on immobile factors, cut spending, or competitively reduce their own capital taxes to attract an inflow of capital. Early models of tax competition focus on the latter option and unequivocally predicted a ‘race to the bottom’, that is, in equilibrium, tax rates on mobile tax bases approach zero.[6]

However, empirical analyses do not find much support for the race-to-the-bottom hypothesis (Hays 2003, 2008; Basinger and Hallerberg 2004). Indeed, ‘taxes on mobile

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[6] See inter alia Wildasin 1989, Zodrow and Mieszkowski 1986, and Frey 1990.

capital continue to be the rule rather than the exception' (Plümper et al. 2007: 6). Effective capital tax rates remain positive and converge to a mean tax rate rather than approaching zero (Hays 2008). Hays explains this observation by arguing that a country's tax mix is crucially determined by its political institutions – with consensus democracies having lower tax rates than majoritarian democracies because their political institutions 'constrain political majorities from choosing transfer maximizing capital tax rates' (Hays 2008: 136).[7] Hays thus accepts that competitive pressures on the capital tax rate exist. In his view, however, the ability of governments to actively engage in tax competition is constrained by the domestic political incentive structure governments face and by capital being imperfectly mobile.

Common wisdom has it that the average effective capital tax rate in OECD countries has declined over time, at least since the abolition of capital controls in the early 1980s. However, as figure 1 shows, while a common trend clearly exists between 1966 and 2000, it is upward rather than downward.[8] Whether or not common shocks also exist is not clear from this figure, but one should keep in mind that OECD countries were affected by two oil price hikes during this period.

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7 Three other theories have been put forward to explain the apparent puzzle of tax rates failing to converge to the low-rate equilibrium predicted by early models. First, Rodrik (1997), Garrett (1998) and Swank and Steinmo (2002) argue that shifting tax revenues to immobile factors, especially to labor, is costly. Second, Basinger and Hallerberg (2004) explain persistently high capital tax rates by the existence of veto-players which prevent some governments from lowering tax rates. Third, Plümper et al. (2007) show that empirical observations are in line with a model in which capital mobility is limited and governments are constrained by voter preferences for low budget deficits and tax fairness.

8 The upper and lower band denote the average tax rate plus and minus one standard deviation, respectively.

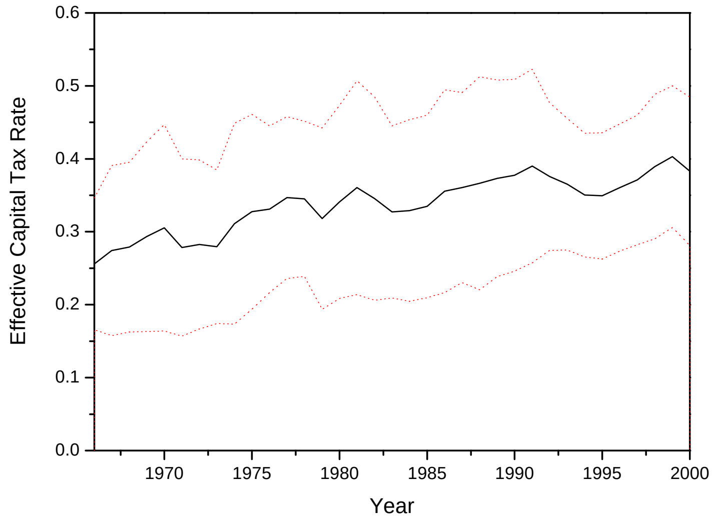

Figure 1: Common Trend in the Effective Capital Tax Rate (Source: Hays 2008).

If they are not fully explained or controlled for, common trends and shocks bias the estimation of spatial lags because when one country has relatively high (low) effective tax rates, the majority of the other countries and thus the weighted mean of the other countries also has relatively high (low) effective capital tax rates even in the absence of spatially dependent tax policies. To demonstrate this, we replicate and extend the analysis of Hays (2008), which builds upon Hays (2003).[9] He analyzes effective capital tax rates in an unbalanced panel of 20 OECD countries over the time period 1966 to 2000. His main variables of interest are capital mobility interacted with various measures of political economy that are of no further interest to us here. In addition to a temporal lag as well as country and period fixed effects, a spatial lag enters the estimations with row-standardized contiguity as the weighting matrix (see next section for a discussion of row-standardization).

[9] Recognizing that a failure to include period dummies may bias the spatial lag coefficient, both Hays (2008) include period dummies, which were missing from Hays (2003).

Table 1: Replication of Hays (2008) and S-OLS Estimation of the Model

	model 1 replication S-ML	model 2 S-OLS with robust s.e.
temporal lag	0.772	0.771
	(0.025) ***	(0.034) ***
spatial lag	0.040	0.047
	(0.010) ***	(0.026) *
capital mobility	0.088	0.088
	(0.038) *	(0.035) *
union density	0.037	0.037
	(0.059)	(0.053)
left government	-0.018	-0.018
	(0.019)	(0.025)
european union	6.670	6.613
	(2.723) *	(3.214)
capital mobility interacted with		
capital endowment	-0.004	-0.004
	(0.001) **	(0.001) ***
consensus democracy	0.016	0.017
	(0.010)	(0.016)
union density	-0.000	-0.000
	(0.000)	(0.001)
left government	0.000	0.000
	(0.000)	(0.000)
european union	-0.074	-0.074
	(0.030) *	(0.035) *
unit fixed effects	yes	yes
period fixed effects	yes	yes
W row-standardized	yes	yes
weight	contiguity	contiguity
R^2	0.921	0.935
Nobs	581	581

Note: results reported in Hays (forthcoming) are not exactly replicable due to minor changes in the data structure. * statistically significant at .1 level ** at .01 level *** at .001 level

Model 1 reported in the first column of table 1 replicates column 2 of table 2 in Hays (2008), using a maximum likelihood (ML) estimator. In the next column we estimate the same model with ordinary least squares (OLS) instead. The coefficient size of the spatial lag variable in model 2 is slightly higher than under ML estimation, but very similar. The standard errors are substantially higher, but the spatial lag is still significantly positive in OLS estimations. In other words, there is evidence for positive spatial dependence: higher tax rates in contiguous countries raise the domestic tax rate and vice versa for lower tax rates. For comparative reasons we stick to the OLS estimations in what follows as some of our estimation models failed to converge with maximum likelihood.

Table 2 presents the estimation results of three models, which deal differently with dynamics, common trends and common shocks. Deviating from Hays's specification, we first exclude the lagged dependent variable from the estimations (model 3), then the period fixed effects (model 4), and then both the lagged dependent variable and the period fixed effects (model 5). Note that the period fixed effects control for common shocks and partly capture common trends (Plümper et al. 2005), while the lagged dependent variable solely but effectively captures common trends and accounts for dynamics.

Table 2: Common Trends and Common Shocks

	model 3 ldv excluded	model 4 period fe excluded	model 5 ldv and period fe ex-cluded
temporal lag		0.772 (0.032) ***	
spatial lag	0.181 (0.044) ***	0.091 (0.028) **	0.372 (0.054) ***
unit fixed effects	yes	yes	yes
period fixed effects	yes	no	no
W row-standardized	yes	yes	yes
weight	contiguity	contiguity	contiguity
R^2	0.835	0.929	0.803
Nobs	581	581	581

Note: all models include the full battery of control variables reported in table 1.

In model 3 the coefficient size of the spatial lag almost doubles whereas the standard error practically stays the same relative to model 2. Clearly then, failure to control for common trends tends to inflate the spatial lag coefficient. Note further that the OLS estimation of model 3 is biased due to serially correlated errors (Beck and Katz 1995). In model 4 the coefficient size of the spatial lag almost quadruples while the standard error only roughly doubles compared to model 2, which suggests that common shocks also bias estimation results for the coefficient of the spatial lag.

In column 5 both period dummies and the temporal lag are left out. The spatial lag now has a coefficient that is almost eight times larger than was the case with period dummies and temporal lag included in the estimations (the aggregate long-term effect is about twice as large). The spatial lag now appears to be far more statistically significant as well as substantively more important. However, this derives from the failure to control for common trends and shocks in model 5. In other words, it is an upward biased result and may lead to wrong inferences.

The results reported so far demonstrate the importance of controlling for common shocks and common trends, especially when the data is so obviously trended as it is for capital taxation. Importantly, for capital taxation we are on safe grounds arguing that the common trend is not caused by the spatial effect, because according to all theories, tax competition should not lead to the common increase in capital taxation, which can be observed in the data, but to a decrease instead. If, however, the common trend is partly due to the spatial lag, then inclusion of period fixed effects or lagged dependent variable can downward bias the coefficient of the spatial lag.

4. Row-Standardization of the Weighting Matrix

While modelling dynamics and controlling for common trends and shocks is an important issue for all panel data, the other two specification problems are peculiar to analyses of spatial dependence as they relate to the weighting matrix. In this section, we deal with whether the weighting matrix should be "row-standardized". This means that for each row of the matrix each cell is divided by its row sum, resulting in a new row-standardized weighting matrix in which the weights in each row now must add up to one.

Our survey of studies employing spatial effects in political science research revealed that few scholars actually row-standardize their weighting matrix (or if they do, they fail to say so). In contrast, spatial econometricians typically treat row-standardization as something that is ‘commonly’ (Franzese and Hays 2006: 174; Franzese and Hays 2008: 29), ‘generally’ (Darmofal 2006: 8) or ‘usually’ (Beck et al. 2006: 28) done. This seems to suggest that row-standardization is both unproblematic and need not be justified.

Neither is warranted. Row-standardization is not unproblematic since, apart from one special case discussed below, it changes the relative weight that observations of all the other (-*i*) units exert in the creation of spatial lags. Thus, it needs to be well justified. Some spatial econometricians are aware of this (e.g., Franzese and Hays 2008: 68; Ward and Gleditsch 2008: 76), but often mention the potential problems of row-standardization merely in passing.

Why would one want to row-standardize at all? One reason given by, for example, Ward and Gleditsch (2008: 76) is that ‘this specific normalization has the advantage that the spatial lag will have the same potential metric or units’ as the dependent variable itself. This can be advantageous if one wants to compare the coefficient size of the spatial lag with that of the temporal lag. However, it is only for one specific type of weighting matrix that row-standardization changes nothing else but the metric or unit of the spatial lag. This specific type is a weighting matrix with unitary weights, which contains values of one in all of the off-diagonal cells. This is in effect identical to not using any weighting at all.

These ‘unweighted’ or ‘identically weighted’ spatial lags are in general unappealing from a theoretical point of view since it is very unlikely that the strength of the contagion effect should be the same independent of the degree with which the ‘infected’ units *i* and the units -*i* from which the spatial effect emanates are connected to each other. For such a weighting matrix row-standardization obviously makes no substantive change. However, for all other matrices row-standardization not only changes the metric or unit of the spatial lag, but also the relative weight given to the -*i* unit observations.

An example helps illustrating this point. Take a weighting matrix that measures contiguity. It has cell entries of one for observations that are contiguous, and zero otherwise. If country *i* has two contiguous countries whereas country *j* has six contiguous countries, then both of *i'*s neighbours and all six of *j'*s neighbours exert the same influence each on the spatial lag variable. After row-standardization, however, the two neighbors of *i* now exert an influence on the spatial lag that is three times larger than the influence of the six neighbors of *j*. Row-standardization has changed the relative substantive weight of units from which the contagion originates. Without row-standardization all contiguous countries exert the same influence no matter how many contiguous countries there are. After row-standardization contiguous countries exert an influence that becomes smaller the larger the number of contiguous countries. This can, but need not, be consistent with the theory of spatial dependence under scrutiny.

To illustrate the effect of row-standardization in our replication exercise and for easy comparison, column 1 of table 3 reports again results from model 2, i.e. the results of the model with period dummies and a temporal lag and row-standardized contiguity as the weighting matrix for the spatial lag. Model 6, reported in the second column of table 3, is identical in its specification with one important exception: this time contiguity is not row-standardized in the weighting matrix.

Table 3: The Effect of Row-standardization

	model 2 (repeated)	model 6 not row-standardized
temporal lag	0.771	0.761
	(0.034) ***	(0.034) ***
spatial lag	0.047	0.020
	(0.026) *	(0.007) **
unit fixed effects	yes	yes
period fixed effects	yes	yes
W row-standardized	yes	no
weight	contiguity	contiguity
R^2	0.935	0.936
Nobs	581	581

Note: all models include the full battery of control variables reported in table 1.

The coefficient size changes as one would expect, but the t-values change even more. Whereas the spatial lag coefficient was barely statistically significant at the 10 per cent level with row-standardized contiguity, it is now significant at the 1 percent level. Changing the relative weight of observation from which spatial dependence emanates has thus the potential to impact inference.

The change in relative weights following from row-standardization is not restricted to a binary weighting matrix that only contains values of one or zero. It equally applies to ordinal or cardinal weighting matrices. If the weights relate to, for example, stocks of foreign direct investment (FDI), then row-standardization implies that it is only differences in relative shares of FDI that matter instead of differences in a country's absolute foreign investment exposure.

Our argument is not that one cannot justify a diminishing influence of contiguous units as the number of these units increases or cannot justify measuring connectivity by FDI stock shares instead of absolute FDI stock exposure. Depending on the context, one clearly can. Rather, our point is that row-standardization is not innocuous. It changes the relative substantive weight of units from which the spatial dependence originates and therefore needs careful theoretical justification. In other words, row-standardization is not just a question of convenience for making the coefficient sizes of the spatial and temporal lags easily comparable and should not be applied as a default rule.

5. The Functional Form of the Weighting Matrix

By far the most popular variables for measuring connectivity in existing spatial econometric work are contiguity and geographical distance (Beck et al. 2006). Apart from the question of row-standardization, it is clear how contiguity is to be specified, namely as a binary matrix with values of one for contiguous units and zero otherwise. However, with non-dichotomous measures such as geographical distance there is no obviously "correct" functional form for specifying connectivity. In many cases, estimation results depend on the assumed functional form, which gives researchers substantial leeway in choosing a functional form that produces results favourable to their hypothesis.

To illustrate the problem, we use geographical distance as the measure of connectivity, but our argument applies equally to other substantive weights such as trade or investment links. Assume a theory predicts that the spatial dependence from more proximate units should be stronger than the dependence from more distant units. This would be in line with what is known as the first "law" of geography: "Everything is related to everything else, but near things are more related than distant things." (Tobler 1970: 236). However, assume further that the theory does not specify the degree with which the spatial dependence decreases as distance increases. This would leave researchers with an infinite number of possibilities for specifying a functional form for the weighting matrix. For example, one could specify proximity as $1/d^n$, where d is distance and n is some positive number greater than zero, as $1/(\ln d)^n$ or as $1-d/d_{\max}$, where d_{max} is maximum observable distance, and so on. Furthermore, one can divide the continuum of distance into several discrete bands, e.g., from 0 to 500 miles, 501 to 1000 miles, etc. By changing the weight one attaches to each band, one changes the relative importance that units falling into one of these bands exert on the spatial lag. One popular choice is to set the weight for one or more of these bands to one and the other ones to zero (Gleditsch and Ward 2000; Murdoch and Sandler 2004). This creates a dichotomous weighting matrix out of the continuous variable distance in which units within a certain distance, say within 1000 miles, all exert the same influence, while units further away do not count at all.

To demonstrate the enormous influence of choosing the functional form of the weighting matrix, we now use geographical distance instead of contiguity for the weighting matrix in our replication example.[10] Both contiguity and distance are compatible with many theories of international tax competition. In fact, if one were to ignore that countries can be geographically close to each other without necessarily being contiguous, then contiguity

10 Data come from Mayer and Zignago (2006), which measures distance between the principal cities of countries weighted by population size.

would merely be an extreme form of distance in which spatial dependence derives only from geographically close countries defined as contiguous countries whereas distant (non-contiguous) countries do not count at all. Using a continuous measure of distance relaxes this strict dichotomy. More proximate countries still matter more than more distant countries. Just how much more depends on the functional form used in the weighting matrix.

In model 7, reported in the first column of table 4, we use $1/d = d^{-1}$ in the weighting matrix, where d is distance between countries. In model 8, reported in the second column of table 4, we use $1/\ln d = (\ln d)^{-1}$ for the weighting matrix instead.

Table 4: The Influence of the Functional Form of the Weights on the Estimation Results

	model 7 1/(distance)	model 8 1/ln(distance)
temporal lag	0.763	0.773
	(0.035) ***	(0.034) ***
spatial lag	5.276	-0.361
	(2.111) *	(0.099) ***
unit fixed effects	yes	yes
period fixed effects	yes	yes
W row-standardized	no	no
weight	d^{-1}	$(\ln d)^{-1}$
R^2	0.936	0.937
Nobs	581	581

Note: all models include the full battery of control variables reported in table 1.

The coefficient of the spatial lag is positive and statistically significant in model 7. Strikingly, it becomes negative and statistically significant in model 8. Thus, a seemingly small change in the functional form chosen for the weighting matrix exerts a large influence on the estimated spatial lag, entirely reverting inferences. Model 7 would suggest that higher taxes in other countries, particularly more proximate ones, *raise* the domestic tax rate. In contrast, model 8 suggests that higher taxes in other countries, again particularly so in more proximate ones, *reduce* the domestic tax rate.

Apparently, the difference in results is driven by the fact that in model 7 the weight given to more distant countries decreases much faster than in model 8. In this sample, more proximate countries tend to have a positive impact on domestic tax rates, whereas more distant countries tend to have a relatively stronger negative impact. With $1/d$ as the functional form for the weighting matrix the positive effect dominates, whereas the negative effect dominates with $1/\ln d$ as the functional form, which gives more distant countries a relatively higher weight.

Not all datasets are equally vulnerable to functional form specification of the weighting matrix and it may not always be possible to find functional forms that lead to once positive once negative estimated coefficients for the spatial lag variable. However, all datasets are to some extent vulnerable to functional form specification. Just how much so is almost impossible to tell for those other than the ones choosing the functional form.

The problem posed by the choice of functional form is amplified by the fact that the correct operationalization and functional form of connectivity must be known (based on theoretical reasoning) by the researcher and the validity of these assumptions cannot be tested. As Beck et al. (2006: 28) state: 'As is done in all spatial econometric works, we assume that the structure of dependence between observations is known by the researcher and not estimated. (…) The assumption that these connectivities are known a priori is both a strong assumption and critical for the methods of spatial econometrics to work.' To demonstrate this, the first column of table 4 repeats model 7, in which $1/d$ was the functional form for the weighting matrix. We now revert distance by subtracting distance from the sum of the minimum and the maximum of distance. The resulting variable – let us call it p for proximity – is one that has the same range (same minimum and maximum) as the distance variable, but is perfectly negatively correlated with it. In model 9, reported in column 2, we use $1/p$ as the functional form. Strikingly, the spatial lag is positive and statistically significant in both models 7 and 9, if only marginally so in the latter.

Table 5: The Effect of Significant Changes of the Weight Matrix

	model 7 repeated 1/(distance)	model 9 1/(distance reversed)
temporal lag	0.763	0.775
	(0.035) ***	(0.034) ***
spatial lag	5.276	1.437
	(2.111) *	(0.084) *
unit fixed effects	yes	yes
period fixed effects	yes	yes
W row-standardized	yes	yes
weight	d^{-1}	p^{-1}
R^2	0.936	0.935
Nobs	581	581

Note: all models include the full battery of control variables reported in table 1.

This result may seem counterintuitive. After all, distance and reversed distance (or proximity) are perfectly negatively correlated with each other. If these were not weighting matrices, but simply explanatory variables entering the estimation model on their own, then their coefficients would be the same, but with opposite signs. However, because they are multiplied with the spatial *y* this no longer holds. Even if the two weights are perfectly negatively correlated with each other, the spatial lags never are. It follows that these two spatial lags can both lead to a statistically significant coefficient with the same sign for the spatial lag.

It would therefore be illegitimate to interpret the spatial lag coefficient as telling us anything on the validity of the weighting matrix. For example, a statistically significant positive spatial lag coefficient with $1/d$ as the weighting matrix does not provide evidence that spatial dependence is correctly modeled as decreasing with the inverse of geographical distance. If we are correct in our belief that $1/d$ is the right specification of the weighting matrix, then a positive and significant coefficient of the spatial lag provides evidence that other countries' policy choices affect domestic policy choices and the more so the closer these countries are to the home country. But our belief in the weighting matrix specification cannot be tested.

Since theory must determine the weighting matrix, no empirical test exists which allows researchers to determine the “correct” functional form of the weighting matrix. For this reason there is no straightforward econometric solution to the apparent arbitrariness in the choice of a functional form. We recommend one of two solutions. The first and ideal one is if researchers provide a better specification of the underlying theory. Some theories will suggest that spatial dependence diminishes very rapidly as distance increases, whereas others would suggest that such dependence diminishes only slowly. Some theories will suggest that spatial dependence diminishes at an increasing, others at a decreasing rate. Admittedly, even with better specified theories some arbitrariness will remain. Still an infinite number of functional forms can specify, say, rapidly decreasing spatial dependence that decreases at an increasing rate. However, more specified theories lead to less arbitariness than less specified ones.

Robustness tests provide the second-best solution. If scholars can show that their results uphold using several functional forms for the weighting matrix and the results are sufficiently similar, then one can be more confident in the existence of a true spatial effect. At the very least, we would suggest testing the robustness of results to such simple modifications of the weighting matrix as taking the natural log of the connectivity variable as well as using its squared value. Converting a continuous connectivity variable into several discrete bands and reporting results for each band separately may also be worthwhile. What is the best way to demonstrate robustness depends on the problem at hand. The important message is that demonstrating robustness is necessary in the absence of a theory that provides sufficient guidance on the functional form.

6. Conclusion

Model specification matters, and even more so in the analysis of spatial dependence. In this article, we have demonstrated that seemingly small changes to the specification of one of Hays's (2003, 2008) models of tax competition lead to a surprisingly large variety of results that are partly contradictory. Our replication exercise raises three important issues that spatial analysts need to address.

First, failure to model dynamics and control for common trends and common shocks will lead to upward bias in the spatial effect estimates. That such failure causes biased coefficient estimates belongs of course to the standard repertoire of knowledge on non-spatial panel data analysis. If anything, it is even more important for getting an unbiased estimate of spatial effects in spatial panel data analysis.

Second, the question of row-standardization must be decided on theoretical grounds and should not be employed as a general default rule. Row-standardization changes the relative weight of observations from which spatial dependence emanates in all weighting matrices but the unitary one, which is a generally unappealing matrix. For all weighting matrices of interest therefore row-standardization will influence the results and may impact inference.

Finally, we have shown that estimation results crucially hinge on the functional form of the weighting matrix, unless the matrix consists of a binary variable such as contiguity. For continuous or categorical variables measuring connectivity, researchers need to be concerned not only about whether or not to row-standardize, but also about choosing the right functional form. As we have demonstrated, small changes to the functional form can lead to very different results. Spatial analysts are thus vulnerable to the charge that they can get the result they wish to obtain by choosing the functional form needed to generate the result.

There are no simple econometric fixes for any of these three problems. Not controlling for common trends and common shocks is likely to lead to upward bias of the spatial lag coefficient. However, controlling for these dynamics by adding period dummies and either the temporally lagged dependent variable as suggested by Beck and Katz (1995), or Prais-Winsten transformation as advocated by Plümper et al. (2005) or by a distributed lag model as preferred by Adolph et al. (2005) may easily lead to the opposite problem. If the trend is partly explained by the spatial lag, then these control mechanisms are likely to lead to downward bias in the estimated coefficient of the spatial lag since it is all too easy for the period dummies (and, if applicable, the temporal lag to fully capture the trend (Plümper at al. 2005). Which bias is more problematic will depend on the context. We recommend that researchers carefully consider different options for modeling dynamics and controlling for common trends and common shocks and that they show how robust the results are to different dynamic modeling options.

We also do not see a straightforward econometric solution to the problem of specification of the weighting matrix. In its absence, we believe it is generally justified to expect researchers to derive predictions on whether to row standardize the weighting matrix (or not) from theory. We are more skeptical whether theories of spatial dependencies will ever be able to convincingly predict a functional form for the weighting matrix. Even then, we believe that researchers can develop their theories further, specifying that certain types of functional forms are more plausible, while others are excluded. For example, in many cases it would seem possible to theoretically justify whether the first and second derivatives of the functional form are positive or negative. For example, a theoretical model should not only be able to tell us that spatial dependence decreases with geographical distance, but also whether it decreases slowly or rapidly and at an increasing or decreasing rate as other units are located further away. In the absence of a theoretically fully specified functional form of the weighting matrix only robustness tests can help. At the least, applied researchers should show whether the spatial effect is robust if they use a linear, a logged and a squared function of the weighting matrix. This may be a good idea in any case even if one is fairly confident that one has specified the functional form on firm theoretical grounds.

The more developed the underlying theory of spatial dependence, the less arbitrary the specification of the empirical model. Of course, it is trivially the case that, all other things equal, a more comprehensively specified theory is better than a less comprehensively specified one. However, this seems to be more important for the analysis of spatial dependencies than in most other fields of research. The peculiar effects of the weighting matrix on the estimation results and the fact that researchers cannot test but have to assume its correctness, make more theoretical guidance an essential element of the research process.

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