Spatial Effects in Dyadic Data

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Abstract

Political units often spatially depend in their political choices on other units. This is no less so in dyadic settings where, as in much of international relations research, the focus of the analysis is the pair or dyad of two political units. Yet, with few exceptions social scientists have analyzed contagion only in monadic datasets, consisting of individual political units. This article explicates and categorizes all possible forms of modeling spatial lags in both undirected and directed dyadic data. This enables scholars to formulate and test novel mechanisms of contagion, thus paving the way for a whole new generation of analyzing spatial dependence between dyads of political units. We illustrate the modeling flexibility gained from an understanding of the full set of specification options for spatial effects in dyadic data by an application to the diffusion of bilateral investment treaties between developed and developing countries, building and extending on a study published in this journal (Elkins et al. 2006). We find that in addition to the target country contagion modeled in the original article, there is also evidence for positive target-to-dyad and source-to-dyad contagion, whereas source country contagion seems to negatively affect the likelihood of signing a bilateral investment treaty.
1. Introduction

Policy choices of one political unit are often not independent of policies implemented in other units, in which case they are said to spatially depend on each other. Recently, social scientists have become very interested in analyzing processes of ‘policy contagion’, ‘policy diffusion’, ‘policy spill-over’ and ‘policy agglomoration’ across jurisdictions. The vast majority of these studies have used a monadic dataset.\(^1\) Almost none have adopted a dyadic framework, i.e. a setting where the unit of analysis is the pair or dyad of two political units, which is particularly important in international relations research.\(^2\)

This is surprising because spatial dependence is very likely to exist in a great many relations between dyads. For example, what one country does in relation to another country will practically always influence as well as be influenced by the relations of some other countries to each other. One of the main reasons for the lack of studies analyzing spatial dependence in a dyadic framework is that political scientists are not aware of the many specification options for modeling such dependence in dyadic data. There is also the secondary and more mundane problem of standard computing capacity seemingly preventing the computation of spatial effects in dyadic datasets of the cross-sectional and time dimensions typically found in political science research.

This paper addresses the first problem by providing a categorization of all possible forms of modeling spatial dependence. We will show that in undirected dyads there is not only the possibility of unit contagion, which is the only option in monadic data, but


also what we call undirected dyad contagion, that is, spill-over from dyad to dyad. In
directed dyads, there is more flexibility still because in such data one can distinguish the
source and the target of dyadic interaction. In total, there are five options. One can
model source and target contagion, which are the two equivalents to unit contagion, but
also model dyadic contagion in the form of source-to-dyad, target-to-dyad and, finally,
directed dyad contagion. A recognition of the various ways in which spatial effects in
dyadic data can be modeled will enable scholars to formulate and test different and
novel mechanisms of spatial dependence, thus facilitating and spurring a whole new
generation of studies analyzing inter-relations between dyads of political units at all
levels of the political system - from the global and international right down to the local.
We also offer, in the appendix, a solution to the secondary problem. Creation of spatial
lags in dyadic data seems to require more computational capacities than standard
personal computers offer. However, these seeming limits on analyzing spatial
dependence in dyadic data can be circumvented by creating the spatial lag for each dyad
or each dyad year separately.
We start by briefly discussing spatial dependence in monadic settings before
categorizing the different and complex ways of modeling spatial effects in dyadic data.
This is followed by an exposition of the various ways of choosing a weighting matrix,
which is also more complex in dyadic than in monadic data. In order to demonstrate the
full potential of modeling spatial dependence in dyadic data, we extend the analysis of
Elkins et al (2006) on the diffusion of bilateral investment treaties (BITs) over the
period 1960 to 2000 by formulating and testing all possible forms of spatial lags. To our
knowledge, their study was the first published article to include spatial lags in a directed
dyadic country sample and it is also commendable for clearly specifying and justifying
their modeling approach. We conclude that Elkins et al. (2006) were right in arguing that the aggregate BIT policy choices of capital-importing countries competing with each other drives BIT diffusion. However, this process of diffusion works through additional dyadic channels not included in their analysis as well and competition among capital-exporting countries via these dyadic channels also matters.

2. **Spatial Dependence in Dyadic Data: A Categorization**

Spatial effects between two jurisdictions occur whenever the marginal utility of one unit depends on the policy choices of at least one other unit (Franzese and Hays 2007: 3). There is no shortage of theories predicting processes of policy diffusion and spill-over across units. Elkins and Simmons (2005), Simmons et al. (2006), and Franzese and Hays (2007: 2), among others, distinguish between coercion (i.e. what Levi-Faur 2005 calls top-down approaches to diffusion), externalities (Simmons and Elkins 2004; Franzese and Hays 2006), competition (Hallerberg and Basinger 1998; Basinger and Hallerberg 2004), cooperation (Genschel and Plümper 1997), learning (Mooney 2001; Meseguer 2005), and emulation (Weyland 2005). All of these theories can also be applied to dyadic frameworks. Moreover, all of these theories are in principle compatible with all of the types of contagion categorized below in the sense that depending on the context and the exact formulation of these theories, each theory can require one or more of these types of contagion.

We will show that dyadic data allows the modeling of far more complex forms of spatial dependence than monadic data. In order to see why this is the case, we start with a brief exposition of spatial effects in monadic data. Then we discuss in detail spatial dependence in dyadic data.
2.1. Spatial Dependence in Monadic Data

In its simplest cross-sectional form without control variables spatial dependence in a monadic dataset can be formulated as:

\[ y_i = \rho W y_{-i} + \varepsilon_i \quad \forall \ i \neq -i. \quad (1) \]

The spatial lag \( W y_{-i} \) consists of two elements, namely what in the following we will refer to as the “spatial \( y \)” (\( y_{-i} \)) and the spatial weighting matrix \( W \).\(^3\) The spatial \( y \) is the contemporaneous or temporally lagged value of the dependent variable in all units other than \( i \). This is multiplied with a \( N \cdot N \) block-diagonal spatial weighting matrix \( W \), which measures the relative connectivity between \( N \) number of units \( i \) and all other units \(-i \ (i \neq -i) \) in the off-diagonal cells of the matrix (the diagonal of the matrix has values of zero). The spatial autoregression parameter \( \rho \) gives the impact of the spatial lag \( W y_{-i} \) on \( y_i \). We will discuss the issue of specifying the weighting matrix in more detail below in section 2.5.

Of course, in reality researchers usually do not estimate the basic model displayed in (1), but add control variables, use a cross-sectional time-series data set, control for serially correlated errors by including the lagged dependent variable as suggested by Beck and Katz (1995) or by Prais-Winsten transformation as advocated by Plümper et al. (2005) and sometimes account for unit fixed effects.\(^4\) To keep the exposition simple, in what follows we will ignore everything that distracts from the modelling of spatial

\(^3\) We would call \( y_i \) the spatially lagged dependent variable, which we regard as the more appropriate term, if Anselin (2003: 159) and others did not use this term for the entire spatial lag \( W y_{-i} \).

\(^4\) The estimation of fixed effects (FE) spatial econometric models is not unproblematic for theoretical and methodological reasons. From a theoretical perspective, theories of policy diffusion usually suggest level effects. Levels, however, are ignored if researchers estimate FE model. From a methodological perspective, the estimation of spatial effects may become extremely inefficient when researchers use FE models (Plümper and Troeger 2007).
dependence itself and therefore employ the most basic cross-sectional setting, always keeping in mind that such a model can and should often be extended to include a time dimension, dynamic modelling, control variables etc. 

In monadic data, spatial lags are very flexible with respect to the specification of the weighting matrix, but they leave researchers no choice on the specification of the spatial y other than $y_{-i}$. Dyadic data allows more flexible specifications of the spatial y. Dyadic data comes in two forms - undirected and directed - and we discuss the different options for spatial lags available in both settings in turn after a brief introduction to the differences between monadic and dyadic data.

2.2. Dyadic Data

Dyads constitute a special case of hierarchically structured data where two individual units form a pair (the dyad) and there is typically variation at both of the individual as well as at the dyadic level. There exist directed and undirected dyadic data. In directed dyadic data, the interaction between two dyad members $ij$ initiates with $i$ and is directed toward $j$. There is a source and a target, an origin and a destination, a sender and a recipient, a giver and a taker, an aggressor and a victim, or some similar directed relationship. For example, in international trade one can distinguish between exporters and importers. In foreign investment one can distinguish between home and host countries. In international migration or remittance flows, there are sending and recipient countries. In inter-state violent conflict, there are aggressor and victim states, and so on.

It is important to note that two dyad members share a directed relationship with each

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5 We will similarly neglect all issues of estimation of spatial lag models, which is complicated by the fact that with inter-dependent units of analysis any spatial dependence model suffers from endogeneity (see Anselin 1988; Ward and Gleditsch 2002; Franzese and Hays 2006, 2007, forthcoming a, forthcoming b). There are also many pitfalls that researchers must avoid in estimation, which we discuss in a separate article (Plümper and Neumayer 2008).
other that can, at least in principle, go both ways, such that member \( i \) is once the source and \( j \) the target in the directed dyad \( ij \) and vice versa for the directed dyad \( ji \). However, it need not go both ways and below we will analyze such an example.

In undirected dyadic data, it is either unclear from the data which of the two dyad members initiated the interaction or this question is theoretically unimportant. One therefore either cannot distinguish between \( ij \) and \( ji \) or does not want to make such a distinction; consequently the dyads \( ij \) and \( ji \) are identical and researchers typically keep only one or the other in their dataset in order to avoid ‘double counting’. A good example is the common use of undirected dyadic data in the international conflict literature (Russett, Oneal and Davis 1998; Gartzke, Li and Boehmer 2001).\(^6\) This can be justified if it is difficult to establish who was the aggressor and who the victim of aggression or if what is of theoretical interest is the mere existence of conflict in a dyad, not who initiated it. Another example for theory driving the choice between undirected and directed dyadic data is the conclusion of a contract. If the contract is voluntarily entered into and if it is of no further interest who was the initiator of the contract, then an undirected dyadic dataset suffices. If, however, one is interested in who initiated the contract or if contract agreement has not been reached voluntarily, such that one actor imposed the contract on another actor, or if the contract means different things to the two contract partners, then this could and in fact should be analyzed with a directed dyadic dataset.

A good example of a case in which contracts mean very different things to the two contracting partners are bilateral investment treaties (BITs). Such treaties grant foreign

\(^6\) Some analyze international conflict in a directed dyadic framework, however. See, for example, Ward, Siverson and Cao (2007).
investors certain rights by limiting the policy autonomy of the government of the country hosting the investment, whereas few, if any, costs are imposed on either the foreign investor or its home government (Guzman 1998; Neumayer and Spess 2005). BITs are entered into voluntarily and it is not always clear who initiated the treaty. Nevertheless, even though in principle BITs are symmetric in that both governments face the same restrictions, in reality the vast majority of BITs are concluded between countries with radically different net foreign investment positions. For the predominantly capital-exporting country the BIT mainly provides rights to its investors with few actual restrictions on its policy autonomy, whereas the predominantly capital-importing country experiences the full restrictions on its policy autonomy (which can still pay off due to the economic benefits of increased inward FDI). Because of the asymmetry of the treaty’s effect it makes sense to analyze the conclusion of BITs in a country dyad dataset that is directed from the capital-exporting to the capital-importing country, as is done by Elkins et al. (2006).

The decision to treat a dyadic relationship as directed or undirected thus requires theoretical justification. Our recommendation is to create a directed dyadic dataset whenever this is possible, given the data at hand, because this allows a more flexible modeling of spatial lags, as we will show below. A directed dataset can easily be used as an undirected dataset by excluding half of the dyads from the estimations, whereas the opposite is not the case. We will start, however, by discussing the modeling of spatial dependence in undirected dyadic datasets since these are more common in the existing literature.

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7 Elkins et al. (2006) argue that capital-importing developing countries are the major initiators of BITs, whereas Neumayer (2006) argues that BITs initiate from capital-exporting developed countries.
2.3. **Modeling Spatial Dependence in Undirected Dyads**

The easiest way of modeling spatial dependence in undirected dyadic data resembles the one used from monadic data analysis. Specifically, researchers can assume that the dependent variable in a dyad is a function of the weighted sum of the dependent variable in all other countries:

\[ y_{ij} = \rho W_{ik}y_k + \varepsilon_{ij}, \tag{3a} \]

which due to the undirectedness of the dyadic dataset is equivalent to

\[ y_{ji} = \rho W_{jk}y_k + \varepsilon_{ji}, \forall \ i \neq j, i \neq k, j \neq k \text{ and } ij = ji. \tag{3b} \]

Equations (3a) and (3b) are appropriate if, for example, one thinks that the decision of whether country \( i \) and country \( j \) enter into a voluntary treaty depends in some form, as specified by the weighting matrix \( W \), on the number of agreements that countries other than either \( i \) or \( j \), called countries \( k \), have concluded. This is the modeling strategy adopted in Manger’s (2006) analysis of the diffusion of preferential trade agreements and in Gleditsch and Gartzke’s (2006) analysis of the effect of alliance ties on international conflict. We call this form of spatial dependence unit contagion.

However, the analysis of undirected dyadic data offers one additional modeling option. Rather than arguing that the spread of a treaty type is merely a function of the number of existing treaties in all other countries, one could argue that this spread is a function of the sum of existing treaties in other dyads, which we call undirected dyad contagion. Thus,

\[ y_{ij} = \rho W_{-i-j}y_{-i-j} + \varepsilon_{ij}, \tag{4a} \]

which again due to the undirectedness of the dyadic dataset is equivalent to
Equations (4a) and (4b) are appropriate if one thinks that, for example, the decision of whether country $i$ and country $j$ should enter into a voluntary agreement depends in some form, as specified by the weighting matrix $W$, on the number of agreements of other dyads, consisting of countries other than $i$ and $j$. As mentioned above, Gleditsch and Gartzke (2006) model unit contagion in their analysis of alliances and international conflict, but they additionally model undirected dyad contagion.

Equations (3a) and (3b) will be different from (4a) and (4b) because the number of treaties between dyads other than $ij$ (other than $ji$) are treated differently. While these dyads are included in the spatial lags of equations (4a) and (4b), they are excluded from (3a) and (3b). In addition, the weighting matrix used in both sets of equations can differ - we leave this issue for discussion in section 2.5.

2.4. **Modeling Spatial Dependence in Directed Dyads**

The number of options for modeling spatial dependence further increases if we analyze directed dyadic data. To recall, in directed dyads, two actors $i$ and $j$ have an asymmetric interaction and one can distinguish units $i$ (the source, origin, sender, giver, aggressor and so on) from units $j$ (the target, destination, recipient, taker, victim and so on).

As with undirected dyadic data, there is the possibility of modeling simple unit contagion, but now there are two options: the directed dyad $ij$ can spatially depend on the actions of other sources $i$ or on the actions of other targets $j$. The first option leads to

$$y_{ji} = \rho W_{y_{-i}} + \epsilon_{ji} \quad \forall \ ij \neq -i - j, \ ji \neq -j - i, \ ij = ji, \ -i - j = -j - i.$$  

(5)
and describes a situation in which contagion derives from the aggregate behavior of the source of the interaction. We name this source contagion. Whereas the second option leads to

\[ y_{ij} = \rho Wy_{-i} + \varepsilon_{ij} \]  

(6)

and denotes a situation in which contagion derives from the aggregate behavior of the target of the interaction, which we call target contagion.

Equation (5) is appropriate if, for example, the number of BITs of other capital-exporters -i are decisive for the decision between a capital-exporting country i and a capital-importing country j on whether to sign a BIT. Whereas (6) is appropriate if the BIT status of other capital-importers -j is decisive.

There are three more possibilities for modeling spatial dependence in directed dyad datasets. They arise when researchers do not take the weighted aggregate of -i or -j units, but the weight of interactions between at least one unit in the dyad ij and one unit outside the dyad ij, namely either -ij, i-j or -i-j. When

\[ y_{ij} = \rho Wy_{-i} + \varepsilon_{ij} \]  

(7)

the probability of, say, two countries i and j signing a BIT depends on the weighted sum of BITs signed by other capital-exporting countries -i with the same capital-importing country j. This we name source-to-dyad contagion. Whereas if

\[ y_{ij} = \rho Wy_{-j} + \varepsilon_{ij} \]  

(8)

then this probability depends on the weighted sum of BITs signed by other capital-importing countries -j with the same capital-exporting country i. This type of spatial dependence we name target-to-dyad contagion.
Finally, the dyad $ij$ can be modeled to be more likely to sign a BIT if other dyads -$i-j$ between capital-exporting and capital-importing countries have already agreed on such a treaty, a form of spatial dependence we call directed dyad contagion. Hence:

$$y_{ij} = \rho Wy_{-i-j} + \epsilon_{ij}. \quad (9)$$

Equation (9) describes a situation in which Thailand and the UK are more likely to sign a BIT if, say, Vietnam and Germany have already signed one.

In addition, one can of course combine all the possible spatial lags of directed dyadic data into

$$y_{ij} = \rho_1 Wy_{-i} + \rho_2 Wy_{-j} + \rho_3 Wy_{-i-j} + \rho_4 Wy_{-j} + \rho_5 Wy_{-i-j} + \epsilon_{ij} \quad (10)$$

Estimating (10) may suffer from multicollinearity unless the weighting matrices for the various spatial lags are sufficiently asymmetric. Clearly, modeling spatial lags is not trivial when the data is dyadic and even more so if we analyze directed dyadic data. With so many options, careful theoretical reasoning is required to rule out certain spatial lags so that (10) can be simplified.

2.5  The Choice of Weighting Matrices

The specification of spatial effects requires two choices. First, the researcher needs to specify the spatial $y$, as discussed above. Second, one also needs to specify the type of weighting matrix used to specify the connectivity between units or dyads that form the spatial dependence. This also requires some careful attention. $^8$

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$^8$ We do not discuss the issue of row standardization here. Row standardization is an important issue (see Plümper and Neumayer 2008) - but it is equally important for monadic and dyadic data. We also stress that the weighting matrix has to be theoretically specified and different weighting matrices cannot be empirically tested against each other (Beck et al. 2006; Plümper and Neumayer 2008). In other words, it has to be assumed that the weighting matrix is correctly specified for spatial econometrics to work.
In monadic data analysis, the weighting matrix provides a link between unit $i$ and other units $-i$. For each observation on $y_j$, the corresponding element of $Wy$ gives a weighted sum of the $y_{-j}$ observations, with weights given by the relative connectivity from $i$ to $-i$, that is, $W = w_{i,-i}$. Hence, the spatial effect is a weighted function of the dependent variable in all other units.

These weighting matrices can be made up of information of different complexity. The simplest choice is to use unitary weights by employing a matrix with values of one in all the off-diagonal cells of the weighting matrix. This is in effect identical to not using any weighting at all since with such a matrix $Wy_{-i} = y_{-i}$. These ‘unweighted’ or ‘identically weighted’ spatial lags are in general theoretically unappealing since it is very unlikely that the strength of the contagion effect should be the same independent of the degree with which the ‘infected’ units and the units from which the spatial effect emanates are connected to each other.

In addition, researchers can employ binary weights by using a dichotomous weighting matrix. The matrix takes on values of one for spatially relevant units and values of zero otherwise. Modeling spatial weights by country contiguity is one example for such a dichotomous weighting matrix. Joint membership in an international organization or a shared culture or civilization are but two other options. Finally, ordinal weights and cardinal weights can account for the intensity of connectivity. Accounting for the intensity of connectivity usually makes theoretical sense since the question of whether a unit exerts a spatial effect will often not be a binary one (yes or no), but a question of how much. According to what is known as the first “law” of geography: “Everything is related to everything else, but near things are more related than distant things.” (Tobler 1970: 236). By far the most common cardinal measure used is geographical distance,
but there are often good reasons to use theoretically-derived, substantive weights instead (see Beck et al. 2006).  

In undirected dyadic data, the choice of a weighting matrix becomes slightly more complicated. In monadic data it is always the connectivity between the unit \(i\) and all other units -\(i\) that enters the matrix in binary, ordinal or cardinal form. For the case of unit contagion in undirected dyadic data the effect of the spatial \(y\) may be weighted by a link function measuring connectivity between either country \(i\) or country \(j\) on the one hand and all other countries \(k \neq \{i, j\}\) on the other hand, i.e., \(W = w_{ik}\) or \(W = w_{jk}\). These link functions can also be applied for the case of undirected dyad contagion, but there are then two additional options available. First, one can combine the connectivity between country \(i\) and other countries \(k\) with the connectivity between country \(j\) and other countries \(k\) in any way. The two simplest ways of combining the two connectivities are linear addition and multiplication. More complicated combinations may be theoretically warranted in certain cases. Second, one can measure the connectivity between dyad \(ij\) and that of other dyads -\(i-j\), which is equal to a link function between dyads \(ji\) and other dyads -\(j-i\) due to the undirectedness of the dyadic data set. This leads to \(W = w_{(ij)(-i-j)}\).

In directed dyadic data, researchers need to think more carefully still about the specification of the weighting matrix. Specifically, in all cases one needs to decide

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9 An ordinal or cardinal weighting matrix may or may not include values of zero to denote certain observations as irrelevant for the spatial effects. Alternatively, one can multiply an ordinal or cardinal weighting matrix that contains only non-zero values with another binary matrix to generate the same effect. More generally, one can combine several weighting matrices of any type with each other. Weighting matrices can also, in principle, take both positive and negative values to account for both positive and negative connectivity. For example, one may theorize that military spending by NATO countries reduced military spending of some contiguous countries (e.g., Austria and Finland) during the Cold War period, while military spending of Warsaw Pact members increased their spending. In these cases, we recommend using two different spatial lags to estimate the effect of positive and negative externalities separately.
whether it is units or dyads that are linked to each other. In source and source-to-dyad contagion, what matters can be either the connectivity between the source unit \( i \) and other source units \(-i\) (\( W = w_{i-} \)) or the connectivity between the dyad \( ij \) and dyads \(-ij\) (\( W = w_{(ij)-} \)), i.e. connectivity between the dyad consisting of source unit \( i \) and target unit \( j \) on the one hand and dyads comprised of other source units \(-i\) and the same target unit \( j \) on the other hand. Similarly, in target and target-to-dyad contagion the link function can either measure connectivity between the target unit \( j \) and other target units \(-j\) (\( W = w_{j-} \)) or measure connectivity between the dyad \( ij \) and dyads \( i-j \) (\( W = w_{(ij)i-j} \)), i.e. dyads of other target units \(-j\) with the same source unit \( i \). Finally, in directed dyad contagion a function combining in any way any of the above mentioned weighting matrices becomes possible. In addition, the link function can measure connectivity between dyad \( ij \) and all other dyads \(-i-j\), which leads to \( W = w_{(ij)(-i-j)} \).

2.6. Summary

In this section, we have categorized all possible forms of spatial dependence along the distinction between monadic data, undirected dyadic data, and directed dyadic data. Monadic data leaves researchers with no choice: a policy in one unit is some weighted function of policies in all other units. If researchers leave the solid ground of monadic data and enter the much more shaky world of multileveled dyadic data, spatial dependence can take several different forms. Table 1 summarizes all the options available for specifying spatial lags (both spatial \( y \) and weighting matrices) in undirected and directed dyadic data.
Table 1. Spatial lag specification in monadic and dyadic data.

<table>
<thead>
<tr>
<th>Type of Contagion</th>
<th>(Partial) Model</th>
<th>Specification of Weighting Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monadic Data:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Contagion</td>
<td>( y_i = \rho W y_{-i} + \epsilon_i )</td>
<td>( W = w_{i-i} )</td>
</tr>
<tr>
<td><strong>Undirected Dyads:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Contagion</td>
<td>( y_{ij} = \rho W y_{ij} + \epsilon_{ij} )</td>
<td>( W = \begin{cases} w_{ik} \ w_{jk} \end{cases} )</td>
</tr>
<tr>
<td>Undirected Dyad Contagion</td>
<td>( y_j = \rho W y_{-j} + \epsilon_j )</td>
<td>( y_i = \rho W y_{-i} + \epsilon_i )</td>
</tr>
<tr>
<td><strong>Directed Dyads:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source Contagion</td>
<td>( y_{ij} = \rho W y_{ij} + \epsilon_{ij} )</td>
<td>( W = \begin{cases} w_{i-j} \ w_{(ij)(-ij)} \end{cases} )</td>
</tr>
<tr>
<td>Target Contagion</td>
<td>( y_{ij} = \rho W y_{ij} + \epsilon_{ij} )</td>
<td>( W = \begin{cases} w_{j-i} \ w_{(ij)(-j)} \end{cases} )</td>
</tr>
<tr>
<td>Source-to-Dyad Contagion</td>
<td>( y_{ij} = \rho W y_{ij} + \epsilon_{ij} )</td>
<td>( W = \begin{cases} w_{i-j} \ w_{(ij)(-j)} \end{cases} )</td>
</tr>
<tr>
<td>Target-to-Dyad Contagion</td>
<td>( y_{ij} = \rho W y_{ij} + \epsilon_{ij} )</td>
<td>( W = \begin{cases} w_{j-i} \ w_{(ij)(-j)} \end{cases} )</td>
</tr>
<tr>
<td>Directed Dyad Contagion</td>
<td>( y_{ij} = \rho W y_{ij} + \epsilon_{ij} )</td>
<td>( W = \begin{cases} w_{j-i} \ w_{(ij)(-j)} \end{cases} )</td>
</tr>
</tbody>
</table>

- \( \epsilon \) represents a random disturbance term.
- \( W \) is the weighting matrix, with \( w \) representing the weights between units.
- \( f \) is a function of any combination of the above weighting matrices.
Clearly, not only does the proper specification of spatial effects become more flexible when researchers move from monadic data to undirected dyadic data and from there to directed dyadic data. Researchers also gain degrees of freedom in the specification of the weighting matrices. In monadic data, only unit contagion is possible and the weighting matrix always specifies connectivity among the monadic units. In undirected dyadic data, undirected dyad contagion is added as an option and the weighting matrix can measure connectivity between one of the dyadic units or both of them with other units or connectivity between the dyad and other dyads as well as combinations of these can be modeled. In directed dyadic data, two types of unit contagion (source and target contagion) and three types of dyad contagion (source-to-dyad, target-to-dyad and directed dyad contagion) are possible. The weighting matrix can measure connectivity between source units, target units or between the dyad and other dyads with the same source unit or the same target unit. In the case of directed dyad contagion, one can model combinations of these or model connectivity between the dyad and other dyads that contain neither the same source nor the same target unit.

In sum, the increased flexibility means that the analysis of spatial effects in dyadic data sets requires much more theoretical reasoning than in monadic data. Researchers need to consider what type of contagion is required by their theoretical model and also justify the specification of the weighting matrix.

3. **Application: Spatial Dependence in a Directed Country Dyad Sample of BIT Diffusion**

In order to demonstrate how the different forms of spatial dependence can lead to different insights on the process of policy diffusion, we build upon and extend Elkins et al.’s (2006) analysis of the diffusion of BITs over the period 1960 to 2000 using a Cox
proportional hazard model. They develop a theory in which this diffusion is driven by competition among potential host countries, typically developing countries, for FDI from typically developed countries.\textsuperscript{10} We restrict our analysis to dyads in which the source country $i$ is a Western developed country and other countries are included as target countries $j$ only, giving us a sample of dimension $(N_i - 1) \cdot (N_j - 1) \cdot T$ in which source countries are never target countries and vice versa.\textsuperscript{11}

Elkins et al. (2006) propose three measures of competition: export market, export product and infrastructure competition. For our application we use export product competition, which we regard as theoretically most plausible. The weighting matrix thus measures the extent to which countries export a similar basket of goods on a scale from -1 to 1 (from total dissimilarity to total similarity).\textsuperscript{12}

Elkins et al. analyze target contagion by estimating a variant of equation (6), namely

$$y_{ijt} = \rho w_{j-1} \sum_{s=1}^{t-1} y_{j-1} + \epsilon_{ijt} \hspace{1cm} (11)$$

In words, the spatial dependence derives from the cumulative sum of existing BITs in other developing countries up to year $t-1$, with connectivity measured by the row-standardized matrix $w_{j-1}$, that is, by export product competition among developing countries. A developing country $j$ is more likely to sign a BIT with a developed country

\textsuperscript{10} In their directed dyadic country sample, the richer country of a dyad is always the origin country $i$ and the poorer country is the destination country $j$ and dyads between high-income countries are excluded.

\textsuperscript{11} Western developed countries are defined as Canada, the US, Western European countries, Japan, Australia and New Zealand. These countries typically do not conclude BITs with each other and FDI flows almost exclusively from developed to developing countries over the period 1960 to 2000.

\textsuperscript{12} Like Elkins et al (2006) we add one to this measure so it runs from 0 to 2.

\textsuperscript{13} They also include control variables of course, which we suppress from the formal exposition for simplicity.
i at time \( t \) if other developing countries -j with a similar basket of export products have signed more BITs with developed countries in the period up to \( t-1 \).

To this, we add all the other possible forms of spatial dependence. We do so in order to show what happens to the results of Elkins et al. (2006), but also because these other forms can be theoretically justified. To start with, we will analyze source diffusion by estimating a variant of equation (5), namely

\[
y_{ijt} = \rho w_{t-1} \sum_{j=1}^{j-1} y_{jlt} + \epsilon_{ijt}.
\] (12)

The only difference to Elkins et al. (2006) is that it is now the aggregate cumulative BITs of developed countries competing in export markets that matters instead of competition among developing countries. Such a specification can be justified if, as suggested by Neumayer (2006), one believes that developed countries are the driving force behind the conclusion of BITs.

Both forms of spatial dependence so far assumed that it is the aggregate behavior of competing developing or developed countries that matters. However, it may well be that countries look more specifically at the question with whom their competitors have signed a BIT, not just how many BITs have been signed by competitors no matter with whom. If such alternative calculation is relevant, then target-to-dyad and source-to-dyad contagion become relevant. With

\[
y_{ijt} = \rho w_{j-1} y_{j-1} + \epsilon_{ijt},
\] (13)

a developing country \( j \) is more likely to conclude a BIT with a developed country \( i \) if other export product competing developing countries -j have previously concluded a BIT with the same developed country \( i \). Whereas with

\[
y_{ijt} = \rho w_{t-1} y_{j-1} + \epsilon_{ijt},
\] (14)
a developed country $i$ is more likely to conclude a BIT with country $j$ if other export product competing developed countries -$i$ have previously concluded a BIT with that same developing country $j$.

Finally, it is conceivable as well that both countries of a dyadic pair $ij$ are influenced by the behavior of other competing developed and other competing developing countries with respect to each other. This leads us to model directed dyad contagion in the form of

$$y_{ijt} = \rho w_{i-i}w_{j-j}y_{i-ijt-1} + \epsilon_{ijt}, \quad (15)$$

where country $i$ is more likely to sign a BIT with country $j$ if other developed countries -$i$ with whom country $i$ competes have previously signed a BIT with other developing countries -$j$ with whom country $j$ competes.$^{14}$

Table 2 presents the estimation results. We include most of the control variables of Elkins et al. (2006) using the same sources of data. To be consistent with their analysis we do not instrument for the spatial lags or apply spatial maximum likelihood even though this may be warranted given serial correlation in the data, in which case the use of spatial lags temporally lagged by one time period cannot fully solve the endogeneity problem. Importantly, we normalize the spatial lags to fall into the interval from 0 to 100 by dividing each lag by its maximum and multiplying by 100. This is because above we have specified target and source contagion as relating to the accumulated number of BITs in other targets and sources, whereas the spatial lags for target-to-dyad,

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$^{14}$ We have chosen a multiplicative functional form connecting the weighting matrix of export product competition among developed countries ($w_{i-i}$) with that of developing countries ($w_{j-j}$). By doing so we implicitly assume that both competitions are simultaneously important for the spatial lag. A linear additive form would have assumed that competition among, say, developing countries can substitute for the lack of competition among developed countries. The results below are hardly affected if one chooses this alternative functional form. As pointed out in section 2.5 above, other combinations are of course also possible.
source-to-dyad and directed dyad contagion all relate to the presence or absence of BITs in other dyads. The spatial lags are therefore held in different units and normalization allows us to compare the substantive importance of all the spatial lags with each other.
Table 2. Spatial Dependence in the Diffusion of Bilateral Investment Treaties.

<table>
<thead>
<tr>
<th></th>
<th>model 1</th>
<th>model 2</th>
<th>model 3</th>
<th>model 4</th>
<th>model 5</th>
<th>model 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>target contagion</td>
<td>source contagion</td>
<td>target-to-dyad contagion</td>
<td>source-to-dyad contagion</td>
<td>directed dyad contagion</td>
<td>multiple forms of contagion</td>
</tr>
<tr>
<td>$w_{i,j} \sum_{t=1}^{T} y_{i,j,t}$</td>
<td>1.006 (0.003) *</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.073 (0.020) ***</td>
</tr>
<tr>
<td>$w_{i,j} \sum_{t=1}^{T} y_{i,j,t}$</td>
<td>1.002 (0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.921 (0.019) ***</td>
</tr>
<tr>
<td>$w_{i,j}y_{i,j-1}$</td>
<td></td>
<td>1.062 (0.003) ***</td>
<td></td>
<td></td>
<td></td>
<td>1.058 (0.004) ***</td>
</tr>
<tr>
<td>$w_{i,j}y_{i,j-1}$</td>
<td></td>
<td></td>
<td>1.029 (0.002) ***</td>
<td></td>
<td></td>
<td>1.032 (0.003) ***</td>
</tr>
<tr>
<td>$w_{i,j}w_{j,i}y_{i,j-1}$</td>
<td></td>
<td></td>
<td></td>
<td>1.005 (0.004)</td>
<td></td>
<td>1.008 (0.006)</td>
</tr>
<tr>
<td>host extractive industries /exports</td>
<td>0.998 (0.002)</td>
<td>0.998 (0.002)</td>
<td>0.997 (0.002) *</td>
<td>0.998 (0.002)</td>
<td>0.998 (0.002)</td>
<td>0.997 (0.002) *</td>
</tr>
<tr>
<td>common law</td>
<td>0.706 (0.076) **</td>
<td>0.699 (0.076) **</td>
<td>0.671 (0.072) ***</td>
<td>0.776 (0.084) **</td>
<td>0.705 (0.076) **</td>
<td>0.711 (0.077) **</td>
</tr>
<tr>
<td>IMF credit (dummy)</td>
<td>1.498 (0.170) ***</td>
<td>1.509 (0.171) ***</td>
<td>1.223 (0.063) ***</td>
<td>1.238 (0.142) *</td>
<td>1.502 (0.170) ***</td>
<td>1.197 (0.138)</td>
</tr>
<tr>
<td>GDP (host) ln</td>
<td>1.258 (0.065) ***</td>
<td>1.258 (0.064) ***</td>
<td>1.453 (0.165) **</td>
<td>1.191 (0.063) **</td>
<td>1.250 (0.064) ***</td>
<td>1.163 (0.062) **</td>
</tr>
<tr>
<td>per capita income (host)</td>
<td>1.000 (0.000)</td>
<td>1.000 (0.000)</td>
<td>1.000 (0.000)</td>
<td>1.000 (0.000)</td>
<td>1.000 (0.000)</td>
<td>1.000 (0.000)</td>
</tr>
<tr>
<td>GDP growth (host)</td>
<td>1.045 (0.011) ***</td>
<td>1.044 (0.010) ***</td>
<td>1.047 (0.011) ***</td>
<td>1.033 (0.010) **</td>
<td>1.044 (0.010) ***</td>
<td>1.035 (0.011) **</td>
</tr>
<tr>
<td>FDI inflow</td>
<td>1.025 (0.016)</td>
<td>1.030 (0.016) *</td>
<td>1.028 (0.016) *</td>
<td>1.021 (0.016)</td>
<td>1.027 (0.016) *</td>
<td>1.026 (0.016) *</td>
</tr>
<tr>
<td>capital account (host) (% of GDP)</td>
<td>3.388 (1.882) *</td>
<td>3.291 (1.818) *</td>
<td>2.648 (1.479) *</td>
<td>2.989 (1.633) *</td>
<td>3.301 (1.818) *</td>
<td>2.262 (1.262) *</td>
</tr>
<tr>
<td>level of democracy</td>
<td>1.010 (0.008)</td>
<td>1.009 (0.008)</td>
<td>1.008 (0.008)</td>
<td>1.007 (0.008)</td>
<td>1.009 (0.008)</td>
<td>1.008 (0.008)</td>
</tr>
<tr>
<td>diplomatic representation (host)</td>
<td>1.007 (0.003) *</td>
<td>1.007 (0.003) *</td>
<td>1.010 (0.003) *</td>
<td>1.006 (0.003) *</td>
<td>1.007 (0.003) *</td>
<td>1.010 (0.003) *</td>
</tr>
<tr>
<td>colonial ties</td>
<td>2.868 (0.685) ***</td>
<td>2.873 (0.688) ***</td>
<td>1.769 (0.436) *</td>
<td>2.912 (0.696) ***</td>
<td>2.869 (0.686) ***</td>
<td>1.692 (0.422) *</td>
</tr>
<tr>
<td>common language</td>
<td>0.892 (0.167)</td>
<td>0.898 (0.168)</td>
<td>1.030 (0.197)</td>
<td>0.921 (0.173)</td>
<td>0.909 (0.171)</td>
<td>1.112 (0.215)</td>
</tr>
<tr>
<td>N</td>
<td>Nobs</td>
<td>2400</td>
<td>38291</td>
<td>2400</td>
<td>38291</td>
<td>2400</td>
</tr>
<tr>
<td>chi²</td>
<td>228.98 ***</td>
<td>224.69 ***</td>
<td>513.92 ***</td>
<td>340.86 ***</td>
<td>226.37 ***</td>
<td>661.45 ***</td>
</tr>
<tr>
<td>-ll</td>
<td>3611.3</td>
<td>3613.4</td>
<td>3468.8</td>
<td>3555.3</td>
<td>3612.6</td>
<td>3395.0</td>
</tr>
</tbody>
</table>

Reported coefficients are hazard ratios. Standard errors in brackets. *, **, *** significant at 10, 1 and 0.1% level, respectively.
In column 1 the spatial lag models target contagion, following the specification in equation (11). The lag is positive and statistically significant, corroborating the finding of Elkins et al. (2006) that the aggregate past behavior of other export competing developing countries \(-j\) matters for the conclusion of a BIT between developed country \(i\) and developing country \(j\). In column 2 the spatial lag models source contagion instead, in line with equation (12). The lag is statistically insignificant, suggesting that the aggregate past behavior of other export competing developed countries does not matter for the agreement on BITs.

In column 3 we move away from aggregate behavior and the spatial lag models target-to-dyad contagion, i.e. specifies contagion from dyads between other developing countries \(-j\) and the developed country \(i\) of the dyad \(ij\), following equation (13). The lag is positive and highly significant with a \(z\)-value of 18.71, suggesting we can be very sure that a developing country is more likely to sign a BIT with a specific developed country if other export competing developing countries have an existing BIT with the same developed country. In column 4 we model source-to-dyad contagion, i.e. contagion from dyads between other developed countries \(-i\) and the developing country \(j\) of the dyad \(ij\), in line with equation (14). The coefficient of this spatial lag, too, is positive and highly statistically significant. This result qualifies the non-significant finding of column 2. The aggregate BIT behavior of other competing developed countries as such does not matter, but if other competing developed countries have signed a BIT with a specific developing country, then this makes it more likely that a developed country will also want to sign a BIT with this developing country. In column 5, we test the hypothesis that contagion may stem from all other dyads (directed dyad contagion), following the spatial lag specification of equation (15). This spatial lag is
statistically insignificant, suggesting that a developing country is not more likely to sign a BIT with a developed country if other export product competing developing countries have previously signed BITs with other export product competing developed countries. Finally, in column 6 we include all possible forms of spatial lags simultaneously. The results are largely consistent with those from the regressions in which each lag was entered separately. In particular, it remains true that there is no evidence for directed dyad contagion as the coefficient of the relevant spatial lag remains statistically insignificant, whereas as before there is evidence for positive target, target-to-dyad and source-to-dyad contagion. The one striking difference is that while the spatial lag modeling source contagion was statistically insignificant on its own, it is now significantly negative in column 6. This suggests that, controlling for other forms of contagion, a developed country is less likely to sign a BIT with a developing country if other export product competing developed countries have previously signed a larger number of BITs with developing countries.

The normalization of spatial lag variables allows us to assess their relative substantive importance. The coefficient sizes suggest that target contagion has the strongest positive impact, followed by target-to-dyad and source-to-dyad contagion. This ranking of relative importance corroborates the argument of Elkins et al. (2006) that competition among developing countries is very important for the diffusion of BITs. However, this does not capture the entire story. Instead, source-to-dyad contagion also matters, if less strongly, and source contagion even has a negative effect on the diffusion of BITs.

4. Conclusion

When policy makers do not only consider constraints in their own political unit, but are influenced by the choices of policy makers in other units, then political scientists have
to address spatial dependence. Such dependence is not restricted to the influence of individual political units on other units. Rather, they occur also in the relations between pairs or dyads of units. Yet, the fast increase in spatial econometric analysis in monadic data is matched with very little spatial research in dyadic data settings. Moreover, the few existing studies typically model spatial effects in dyadic data very similar to their monadic data counterparts.

This article has demonstrated that spatial analyses can be fruitfully extended to dyadic relations between countries or other units. In particular, we have shown that the menu of choice increases by moving from monadic to undirected dyadic data and increases further still when analyzing directed dyadic data. We have illustrated the modeling possibilities in directed dyadic data by extending Elkins et al.’s (2006) analysis of the diffusion of bilateral investment treaties. In addition to the target country contagion already considered in the original estimations, we found evidence for source country, target-to-dyad and source-to-dyad contagion, but none for directed dyad contagion. Of course, rather than mining the data for all possible types of contagion, researchers should instead start with specifying the types of contagion implied by their theory and test these against the data. We merely wanted to demonstrate the full range of modeling choices.

By carefully specifying the spatial y and the weighting matrices, researchers can study a large variety of possible spatial effects, with each one being based on a different theoretical model. An understanding of the full set of options of specifying spatial effects in dyadic data will allow formulating and testing novel hypotheses predicting dependence of the interaction of two political units on the policy choices of other units.
or other dyads. It will render possible a new generation of research into spatial dependence in dyadic data.

Appendix. Overcoming limits of computing capacity

An important difference between dyadic and monadic datasets is the size of the weighting matrix and spatial $y$ variables. The size of the dataset required to compute spatial lags in dyadic data can quickly become so large as to seemingly exceed computing capacity of standard computers. This appendix demonstrates a computationally viable strategy, which allows generating spatial weights even in large data sets and thus helps researchers overcoming the limits of computing capacity.

To appreciate the problem, recall that in monadic settings, the weighting matrix and spatial $y$ variables always have dimension $N \cdot N \cdot T$, where $N$ is the number of monadic units and $T$ the number of time periods. In undirected dyadic settings, one needs instead a dataset in which each dyad is related to every other dyad in every time period. The weighting matrix and spatial $y$ variables have thus dimension

$$
(N-1) \cdot (N-1) \cdot (N-1) \cdot (N-1) \cdot T / 2,
$$

where $N$ is now the number of units forming dyads with other units.\(^{15}\)

With directed dyad data, the dimension of these variables is

$$
(N_i - 1) \cdot (N_j - 1) \cdot (N_i - 1) \cdot (N_j - 1) \cdot T
$$

---

\(^{15}\) It is possible to include intra-unit interactions in dyadic data, such that the dyad $ii$ is included in the dataset. In this case the dimension of the weighting matrix and spatial $y$ variables is $N \cdot N \cdot N \cdot N \cdot T / 2$. 

where \( N_i \) is the number of origin units, \( N_j \) is the number of destination units.\(^{16}\) This simplifies to

\[
(N - 1) \cdot (N - 1) \cdot (N - 1) \cdot (N - 1) \cdot T
\]

if all origin units can be destination units as well and vice versa.

Clearly, such datasets can become very large indeed for values of \( N \) and \( T \) typically encountered in social science research. For example, if one were to have a directed dyadic sample of 150 countries over a 20 year period, then the dataset needed for constructing a dyadic spatial lag would contain \( 149 \cdot 149 \cdot 149 \cdot 149 \cdot 20 = 9,857,688,020 \) observations, i.e. almost ten billion observations. In addition, the dataset would need to contain at least two dyad identifying variables, a time variable as well as both the weighting matrix and the spatial \( y \) variables. In Stata®, the most widely used econometrics package, such a dataset would require memory size above 100 GB, i.e. orders of magnitude higher than what standard computers offer.

The solution to this problem lies in designing a loop that constructs the spatial lag for each dyad or, depending on the size of the sample, even for each dyad time period separately and then stacking the resulting spatial lag sub-components on top of each other. Upon publication, we will make Stata® code available that can be used by researchers to construct all possible forms of spatial dependence.

\(^{16}\) \( N_i \cdot N_j \cdot N_i \cdot N_j \cdot T \) if intra-unit interactions and therefore dyads \( ii \) are included.
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